Calculus Warm Up #1-4

$x^2 + y^2 = r^2$ $\tan\theta = \frac{y}{x}$

$$\boxed{\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}}$$

Find the slope of the polar curve at the following points:

$(2, 0)$, $(4, \frac{3\pi}{2})$, $(3, \frac{7\pi}{6})$

$r = 2(1 - \sin\theta)$

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In Exercises 3–6, find dy/dx and the slope of the graph of the polar curve at the given value of θ.

Polar Equation	Value of θ
3. $r = 3(1 - \cos\theta)$	$\theta = \frac{\pi}{2}$
4. $r = 3 - 2\cos\theta$	$\theta = 0$
5. $r = 3\sin\theta$	$\theta = \frac{\pi}{3}$
6. $r = 4$	$\theta = \frac{\pi}{4}$

$\frac{dy}{dx} = \frac{-(4\cos^2\theta - 3\cos\theta - 2)}{\sin\theta(4\cos\theta - 3)}$

slope is undefined @ $\theta = 0$

$\frac{dy}{dx} = -\cot\theta$

@ $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -1$

In Exercises 7 and 8, find the points of horizontal and vertical tangency (if any) to the polar curve.

7. $r = 1 + \sin\theta$

8. $r = a\sin\theta$

Horizontal tangents @

$(0,0)$ $(0,\pi)$ $\left(a, \frac{\pi}{2}\right)$; $\left(-a, \frac{3\pi}{2}\right)$

Vertical tangents @

$\left(\frac{a\sqrt{2}}{2}, \frac{\pi}{4}\right)$ $\left(\frac{a\sqrt{2}}{2}, \frac{3\pi}{4}\right)$

$\left(-\frac{a\sqrt{2}}{2}, \frac{5\pi}{4}\right)$ $\left(-\frac{a\sqrt{2}}{2}, \frac{7\pi}{4}\right)$

In Exercises 9 and 10, find the points of horizontal tangency (if any) to the polar curve.

9. $r = 2\csc\theta + 3$

In Exercises 9 and 10, find the points of horizontal tangency (if any) to the polar curve.

9. $r = 2\csc\theta + 3$

$r' = -2\csc\theta\cot\theta$

$\frac{dy}{dx} = \frac{(2\csc\theta+3)\cos\theta + (-2\csc\theta\cot\theta)\sin\theta}{\quad}$

$0 = \frac{2}{\sin\theta}\cdot\cos\theta + 3\cos\theta - \frac{2\cancel{\sin\theta}\cot\theta}{\cancel{\sin\theta}}$

$0 = 2\cot\theta - 2\cot\theta + 3\cos\theta$

$0 = 3\cos\theta$

13.3 Vector Valued Functions

$\boxed{\mathbf{r}(t) = f(t)\,\mathbf{i} + g(t)\,\mathbf{j}}$

f(t) and g(t) are component functions of the parameter t.

$$\boxed{\lim_{t\to a}\left(\mathbf{r}(t)\right) = \left(\lim_{t\to a} f(t)\right)\mathbf{i} + \left(\lim_{t\to a} g(t)\right)\mathbf{j}}$$

(as long as both limits exist)

$$\boxed{\mathbf{r}'(t) = f'(t)\,\mathbf{i} + g'(t)\,\mathbf{j}}$$

Find the derivative fo the vector valued function:

$\mathbf{r}(t) = (4t^2 - 5t)\ i + (6t)\ j$

$r'(t) = (8t-5)i + 6j$

Another notation: $\langle i, j \rangle$

Example: $\mathbf{r}(t) = \langle t - 3, 7t \rangle$

$r'(t) = \langle 1, 7 \rangle$

The curve of a vector valued function, **r**(t), is considered smooth on an open interval (a, b) if:

1) f´ and g´ are continuous on (a, b)

2) $\mathbf{r}'(t) \neq 0$ anywhere on (a, b)

Example:
Find the open interval(s) where **r**(t) is smooth

$\mathbf{r}(t) = \frac{1}{t-1} i + 3t\ j$ $\quad f'(t) = -\frac{1}{(t-1)^2}$ $\quad g'(t) = 3$

$r'(t) = -\frac{1}{(t-1)^2} i + 3j$

discontinuity @ $t=1$

$r'(t) \neq 0$ anywhere.

curve is smooth on $(-\infty, 1) \cup (1, \infty)$

HW:

p. 755 # 11 - 15 odd,

25 - 30, 33 - 37 odd

(Answers to evens follow)

26) $r'(t) = -\dfrac{1}{t^2} i - \dfrac{2}{(t-1)^2} j$

$r''(t) = \dfrac{2}{t^3} i + \dfrac{4}{(t-1)^3} j$

28) $r'(t) = \langle \sec t \tan t, \sec^2 t \rangle$

$r''(t) = \langle \sec^3 t + \sec t \tan^2 t, 2\sec^2 t \tan t \rangle$

30) $r'(t) = \langle -\csc^2 t, 2\cos 2t \rangle$

$r''(t) = \langle 2\csc^2 t \cot t, -4\sin 2t \rangle$